HOMEWORK 6 - ANSWERS TO (MOST) PROBLEMS

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Section 3.4: The Chain Rule

3.4.9. $F'(x) = \frac{1}{4}(1+2x+x^3)^{-\frac{3}{4}}(2+3x^2)$

3.4.15. $y' = e^{-kx} - kxe^{-kx}$

3.4.39. $f'(t) = \sec^2(e^t) + e^{\tan(t)}\sec^2(t)$

3.4.45. $y' = -\sin(\sqrt{\sin(\tan(\pi x))})\frac{1}{2\sqrt{\sin(\tan(\pi x))}}\cos(\tan(\pi x))\sec^2(\pi x)\pi$

3.4.49. $y' = \alpha e^{\alpha x}\sin(\beta x) + e^{\alpha x}\beta\cos(\beta x)$; $y'' = e^{\alpha x}((\alpha^2 - \beta^2)\sin(\beta x) + 2\alpha\beta\cos(\beta x))$

3.4.63.

(a) $h'(1) = f'(g(1))g'(1) = f'(2)g'(1) = 5 \times 6 = 30$
(b) $H'(1) = g'(f(1))f'(1) = g'(3)f'(1) = 9 \times 4 = 36$

3.4.66.

(a) $h'(2) = f'(f(2))f'(2) = f'(1)f'(2) = -\frac{1}{2} \times 1 = -\frac{1}{2}$
(b) $g'(2) = f'(4)4 = 1 \times 4 = 4$

3.4.70.

(a) $f'(x) = g(x^2) + xg'(x^2)2x = g(x^2) + 2x^2g'(x^2)$
(b) $f''(x) = g'(x^2)(2x) + 4xg'(x^2) + 2x^2g''(x^2)2x = 6xg'(x^2) + 4x^3g''(x^2)$

3.4.83. $a(t) = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \frac{dv}{ds}v(t)$

3.4.84. $V(t) = \frac{4}{3}\pi r^3$, so $\frac{dV}{dt} = \frac{4}{3}\pi 3r^2\frac{dr}{dt} = 4\pi r^2\frac{dr}{dt}$

3.4.90.

$$\begin{aligned}(f(x)[g(x)]^{-1})' &= f'(x)[g(x)]^{-1} + f(x)(-1)[g(x)]^{-2}g'(x)\\ &= f'(x)g(x)[g(x)]^{-2} - f(x)g'(x)[g(x)]^{-2}\\ &= \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}\end{aligned}$$

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Section 3.5: Implicit differentiation

3.5.3. $y' = -\frac{y^2}{x^2}$

3.5.19. $y' = \frac{e^y \sin(x) + \cos(xy) y}{e^y \cos(x) - x\cos(xy)}$

3.5.27. $y = x + \frac{1}{2}$

3.5.28. $(y-1) = \sqrt{3}(x + 3\sqrt{3})$, or $y = \sqrt{3}x + 10$

3.5.30. $y = -2$

3.5.40. See separate document 'Solution to 3.5.40'

3.5.46. $y' = \frac{1}{2\arctan(x)} \times \frac{1}{1+x^2}$

3.5.49. $G'(x) = -\frac{x}{\sqrt{1-x^2}} \arccos(x) - 1$

3.5.57. See separate document 'Solution to 3.5.57'

3.5.67.

(a) $f(f^{-1}(x)) = x$, let $y = f^{-1}(x)$, then $f(y) = x$, so $f'(y)y' = 1$, so $y' = \frac{1}{f(y)} = \frac{1}{f(f^{-1}(x))}$

(b) $\frac{3}{2}$

3.5.69. See separate document 'Solution to 3.5.69'